Lec 6: 02/04/2010

Observational evidence for expansion:

The question is how we can find out if the universe is expanding. More precisely, to find the expansion rate at the present time (zero indicates a static universe).

In a FRW universe, the physical distance between the observer ($r=0$) and an object (radial coordinate r) is given by $a(t)r$. Hence, the further the object is (larger r) the faster the distance will increase.

To measure the expansion rate of the universe, we need to measure the distance and the rate at which the distance increases. Redshift is a good measure of the expansion. As we discussed, the redshift of an object emitting photons is given by:

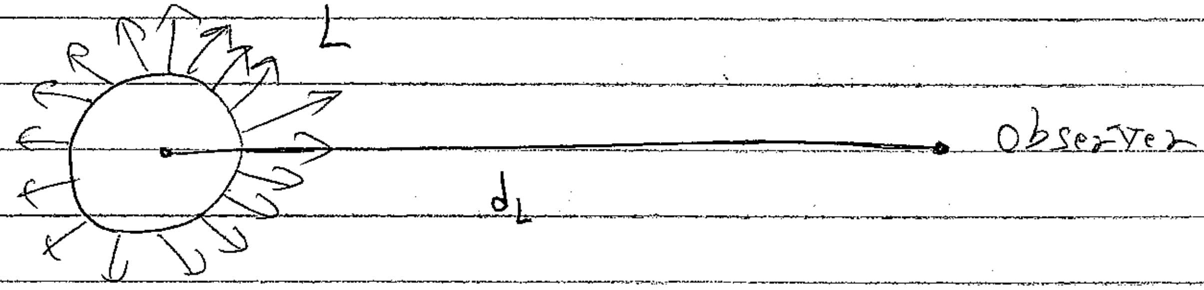
$$1+z = \frac{a(t_0)}{a(t)}$$

where t_0 is the present time, t is the time photons were emitted, and $a_{(t_0)}$; $a_{(t)}$ are the scale factors of the Universe at these epochs.

There are various measures for the distance of an object. Here we mention three relationships through which we can measure the Hubble expansion rate.

(1.) Luminosity distance - redshift relationship.

Consider an object with intrinsic luminosity "L", which is defined as radiate energy per unit time



An observer far away from the source measure flux "F" of energy per unit area per unit time. In the case the energy is not absorbed on its way from the source to the observer, we expect that $F \propto d^{-2}$ where d is the distance between the source and the observer. In general, we define the luminosity distance d_L according to:

$$d_L^2 \equiv \frac{L}{4\pi F}$$

If we know the intrinsic luminosity, we can find d_L by measuring F. In a static universe d_L is just the physical distance between the source and the observer.

Now consider an expanding universe. Looking at nearby objects (small radial coordinate r) ensures that there is no sensitivity to the geometry of the universe. The physical distance at the present time between the observer and the source is:

$$\begin{cases} a_{(t_0)}\, r & k=0 \\ a_{(t_0)}\, \sin r & k=+1 \\ a_{(t_0)}\, \sinh r & k=-1 \end{cases}$$

For small r it simply is $a_{(t)}\, r$ in all the three cases.

The luminosity distance in this case is not just the physical distance $a_{(t_0)}\, r$ (which is the distance light has travelled from the source to the observer). The observed flux (number of photons per unit time per unit area) is redshifted as $(1+z)^2$. One factor of $1+z$ is due to the frequency redshift of each photon, and another $1+z$ factor is because of the redshift in arrival times of two successive photons. Therefore:

$$d_L^2 = a_{(t_0)}^2\, r^2\, (1+z)^2 \Rightarrow d_L = a_{(t_0)}\, r\, (1+z) \quad (*)$$

Now we should find r as a function of

redshift z. The equation for null geodesic along which light travels from the source to the observer is:

$$\int_{t}^{t_0} \frac{dt'}{a(t')} = r \quad (**)$$

Using the Taylor's expansion of $a(t')$, and keeping terms up to second order in $t_0 - t'$, we have:

$$a(t') \simeq a(t_0) + \dot{a}'(t_0)\,(t' - t_0) + \frac{1}{2}\ddot{a}(t_0)\,(t' - t_0)^2 \Rightarrow$$

$$\frac{a(t')}{a(t_0)} \simeq 1 + H_0\,(t' - t_0) - \frac{1}{2} q_0 H_0^2\,(t' - t_0)^2 \quad (***)$$

Where:

$$H_0 \equiv \frac{\dot{a}}{a}\Big|_{t=t_0} \qquad q_0 \equiv -\frac{\ddot{a}}{a}\Big|_{t=t_0} H_0^{-2}$$

"q_0" is the deceleration parameter of the Universe ($q_0 > 0$ for decelerating expansion).

Note that:

$$\frac{a(t_0)}{a(t)} = 1 + z \Rightarrow z \simeq H_0\,(t_0 - t) + H_0^2\left(1 + \frac{q_0}{2}\right)(t_0 - t)^2$$

Using the expansion in (***) and performing the integral in (**), we find:

$$r \simeq \frac{H_0^{-1}}{a(t_0)}\left[z - \frac{1}{2}(1+q_0)z^2\right]$$

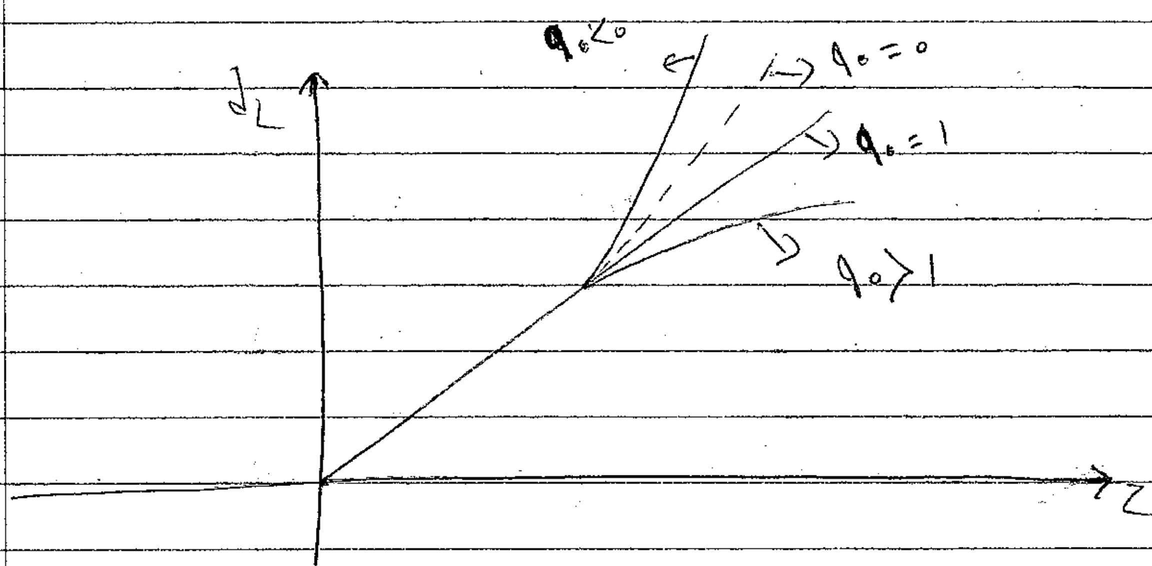
Then (*) results in:

$$d_L \simeq H_0^{-1}\left[z + \frac{1}{2}(1-q_0)z^2\right] \Rightarrow$$

$$\boxed{H_0\, d_L \simeq z\left[1 + \frac{1}{2}(1-q_0)z\right]}$$

This is the so called Hubble's law. For very small z, $d_L - z$ plot is a straight line whose slope is H_0^{-1}. Thus, by measuring the redshift of an object and its observed flux, we can find the ratio $\frac{d_L}{z}$ and derive H_0^{-1}.

If we go to higher z such that the second term on the right-hand side of Hubble's law becomes important, we can also find the deceleration parameter at the present time q_0. The $d_L - z$ diagram (up to second order in z) looks like:



However, to find the luminosity distance from the observed flux, we need to know the intrinsic luminosity, L (recall that $d_L^2 = \frac{L}{4\pi F}$). We therefore need to use "Standard Candles" for which the intrinsic luminosity is known.

It turns out that we need more than one set of objects as standard candles for different values of z. At very small z, variable stars are good candles for which there exists an empirical relation between luminosity and period. However, this will not remain at higher z. Note that higher z means further away, and hence earlier. This means younger galaxies when compared to galaxies at the present time, and hence evolution effects can become important. This makes the use of very close objects for those at larger z unreliable.

Another set of standard candles, which are more reliable at larger z, is type Ia supernova. In fact there was observation of these supernova at high redshift ($z \sim 0.3 - 1$) that give rise to evidence for an accelerated expansion of the universe at recent times. Note that to extract information about q_0 we need to go to higher z (see figure on page 50).

Accelerated expansion ($q_0 < 0$) cannot be obtained from a perfect fluid with $p = 0$ (matter) or $p = \frac{1}{3}\rho$ (radiation). As we discussed, the 2nd Friedmann equation requires that $p + \frac{1}{3}\rho < 0$ in order to have $\ddot{a} > 0$. This implies that $p < -\frac{1}{3}\rho$, hence negative pressure).

A cosmological constant can give rise to accelerate expansion. Recall that the 1st Friedmann equation reads:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{1}{3}\Lambda \quad (\text{assume } k = 0)$$

in the presence of a cosmological constant Λ.

Assuming that $\frac{1}{3}\Lambda$ dominates the right hand side, we find;

$$\frac{\dot{a}}{a} = \sqrt{\frac{1}{3}\Lambda} \Rightarrow a_{(t)} = a_0 \exp\left[H_0(t-t_0)\right]$$

$$H_0 = \sqrt{\frac{\Lambda}{3}}$$

It is easy to see that:

$$q_0 = -\frac{\ddot{a}a}{\dot{a}^2} = -1$$

in this case. For a cosmological constant we have $p = -\rho$, which clearly satisfies the condition $p < -\frac{1}{3}\rho$ for accelerated expansion.